January 2009
6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
1	$x-3$ is a factor	B1
	$\text{f}(x) = (x-3)(2x^2-2x+1)$	M1 A1
	Attempt to solve quadratic i.e. $x=\dfrac{2\pm\sqrt{4-8}}{4}$	M1
	$x=\dfrac{1\pm \text{i}}{2}$	A1 **[5]**

Notes:

First and last terms in second bracket required for first M1
Use of correct quadratic formula for their equation for second M1

Question Number	Scheme	Marks
2 (a)	$6\sum r^2 + 4\sum r - \sum 1 = 6\frac{n}{6}(n+1)(2n+1) + 4\frac{n}{2}(n+1), -n$	M1 A1, B1
	$= \frac{n}{6}(12n^2 + 18n + 6 + 12n + 12 - 6)$ or $n(n+1)(2n+1) + (2n+1)n$	M1
	$= \frac{n}{6}(12n^2 + 30n + 12) = n(2n^2 + 5n + 2) = n(n+2)(2n+1)$ *	A1 (5)
(b)	$\sum_{r=1}^{20}(6r^2 + 4r - 1) - \sum_{r=1}^{10}(6r^2 + 4r - 1) = 20\times 22\times 41 - 10\times 12\times 21$	M1
	$= 15520$	A1 (2) **[7]**

Notes:

(a) First M1 for first 2 terms, B1 for $-n$
Second M1 for attempt to expand and gather terms.
Final A1 for correct solution only

(b) Require (r from 1 to 20) subtract (r from 1 to 10) and attempt to substitute for M1

Question Number	Scheme	Marks
3 (a)	$xy = 25 = 5^2$ or $c = \pm 5$	B1 (1)
(b)	*A* has co-ords (5, 5) and *B* has co-ords (25, 1)	B1
	Mid point is at (15, 3)	M1A1 (3) **[4]**

Notes:

(a) $xy = 25$ only B1, $c^2 = 25$ only B1, $c = 5$ only B1

(b) Both coordinates required for B1
Add theirs and divide by 2 on both for M1

Question Number	Scheme	Marks
4	When n = 1, LHS = $\frac{1}{1\times 2}=\frac{1}{2}$, RHS = $\frac{1}{1+1}=\frac{1}{2}$. So LHS = RHS and result true for n = 1	B1
	Assume true for n = k; $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$ and so $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$	M1
	$\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$	M1 A1
	and so result is true for n = k + 1 (and by induction true for $n\in\mathbf{Z}^+$**)**	B1 **[5]**

Notes:

Evaluate both sides for first B1
Final two terms on second line for first M1
Attempt to find common denominator for second M1.
Second M1 dependent upon first.
$\frac{k+1}{k+2}$ for A1
'Assume true for $n=k$ 'and 'so result true for $n=k+1$' and correct solution for final B1

Question Number	Scheme	Marks
5 (a)	attempt evaluation of f(1.1) and f(1.2) (– looking for sign change)	M1
	f(1.1) = 0.30875 , f(1.2) = -0.28199 Change of sign in f(x) $\Rightarrow$ root in the interval	A1 (2)
(b)	$\mathrm{f}'(x) = \frac{3}{2}x^{-\frac{1}{2}} - 9x^{-1\frac{1}{2}}$	M1 A1 A1 (3)
(c)	f (1.1) = 0.30875.. f ′(1.1) = -6.37086…	B1 B1
	$x_1 = 1.1 - \frac{0.30875...}{-6.37086..}$	M1
	= 1.15(to 3 sig.figs.)	A1 (4) **[9]**

Notes:

(a) awrt 0.3 and -0.3 and indication of sign change for first A1
(b) Multiply by power and subtract 1 from power for evidence of differentiation and award of first M1
(c) awrt 0.309 B1and awrt -6.37 B1 if answer incorrect
Evidence of Newton-Raphson for M1
Evidence of Newton-Raphson and awrt 1.15 award 4/4

Question Number	Scheme	Marks
6	At n =1, $u_n = 5\times 6^0 + 1 = 6$ and so result true for n = 1	B1
	Assume true for n = k; $u_k = 5\times 6^{k-1} + 1$,and so $u_{k+1} = 6(5\times 6^{k-1} + 1) - 5$	M1, A1
	$\therefore u_{k+1} = 5\times 6^k + 6 - 5$ $\quad \therefore u_{k+1} = 5\times 6^k + 1$	A1
	and so result is true for n = k + 1 and by induction true for $n \geq 1$	B1
		[5]

Notes:
6 and so result true for n = 1 award B1
Sub u_k into u_{k+1} or M1 and A1 for correct expression on right hand of line 2
Second A1 for $\therefore u_{k+1} = 5\times 6^k + 1$
'Assume true for n = k' and 'so result is true for n = k + 1' and correct solution for final B1

Question Number	Scheme	Marks
7 (a)	The determinant is a - 2	M1
	$\mathbf{X}^{-1} = \frac{1}{a-2}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$	M1 A1 (3)
(b)	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1
	Attempt to solve $2 - \frac{1}{a-2} = 1$, or $a - \frac{a}{a-2} = 0$, or $-1 + \frac{1}{a-2} = 0$, or $-1 + \frac{2}{a-2} = 1$	M1
	To obtain a =3 only	A1 cso (3) **[6]**
	Alternatives for (b) If they use $\mathbf{X}^2 + \mathbf{I} = \mathbf{X}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1 If they use $\mathbf{X}^2 + \mathbf{X}^{-1} = \mathbf{O}$, they can score the B1then marks for solving If they use $\mathbf{X}^3 + \mathbf{I} = \mathbf{O}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1	

Notes:

(a) Attempt *ad*-*bc* for first M1

$\frac{1}{\det}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$ for second M1

(b) Final A1 for correct solution only

Question Number	Scheme	Marks
8 (a)	$\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $2y\frac{dy}{dx} = 4a$	M1
	The gradient of the tangent is $\frac{1}{q}$	A1
	The equation of the tangent is $y - 2aq = \frac{1}{q}(x - aq^2)$	M1
	So $yq = x + aq^2$ *	A1 (4)
(b)	R has coordinates (0, aq)	B1
	The line l has equation $y - aq = -qx$	M1A1 (3)
(c)	When $y = 0$ $x = a$ (so line l passes through (a, 0) the focus of the parabola.)	B1 (1)
(d)	Line l meets the directrix when $x = -a$: Then $y = 2aq$. So coordinates are ($-a$, $2aq$)	M1:A1 (2) **[10]**

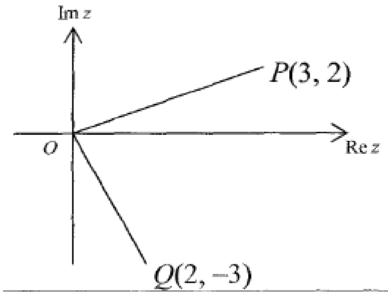
Notes:

(a) $\frac{dy}{dx} = \frac{2a}{2aq}$ OK for M1
Use of $y = mx + c$ to find c OK for second M1
Correct solution only for final A1

(b) -1/(their gradient in part a) in equation OK for M1

(c) They must attempt $y = 0$ or $x = a$ to show correct coordinates of R for B1

(d) Substitute $x = -a$ for M1.
Both coordinates correct for A1.

Question Number	Scheme	Marks
9 (a)	$z_2 = \frac{12-5i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{36-24i-15i-10}{13}$	M1
	$= 2-3i$	A1 (2)
(b)		
	P: B1, Q: B1ft	B1, B1ft (2)
(c)	grad. $OP \times$ grad. $OQ = \frac{2}{3} \times -\frac{3}{2}$	
	$= -1 \quad \Rightarrow \angle POQ = \frac{\pi}{2}$ (✱)	
OR	$\angle POX = \tan^{-1}\frac{2}{3}$, $\angle QOX = \tan^{-1}\frac{3}{2}$	
	$\text{Tan}(\angle POQ) = \frac{\frac{2}{3}+\frac{3}{2}}{1-\frac{2}{3}\times\frac{3}{2}}$ M1	M1
	$\Rightarrow \angle POQ = \frac{\pi}{2}$ (✱) A1	A1 (2)
(d)	$z = \frac{3+2}{2} + \frac{2+(-3)}{2}i$	M1
	$= \frac{5}{2} - \frac{1}{2}i$	A1 (2)
(e)	$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$	M1
	$= \frac{\sqrt{26}}{2}$ or exact equivalent	A1 (2) **[10]**

Notes:

(a) $\times \frac{3-2i}{3-2i}$ for M1

(b) Position of points not clear award B1B0

(c) Use of calculator / decimals award M1A0

(d) Final answer must be in complex form for A1

(e) Radius or diameter for M1

Question Number	Scheme	Marks
10 (a)	**A** represents an enlargement scale factor $3\sqrt{2}$ (centre O)	M1 A1
	B represents reflection in the line $y = x$	B1
	C represents a rotation of $\frac{\pi}{4}$, i.e.45° (anticlockwise) (about O)	B1 (4)
(b)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$	M1 A1 (2)
(c)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$	B1 (1)
(d)	$\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix}$ so (0, 0), (90 , 0) and (51 , 75)	M1A1A1A1 (4)
(e)	Area of $\Delta\, OR'S'$ is $\frac{1}{2} \times 90 \times 75 = 3375$	B1
	Determinant of **E** is –18 or use area scale factor of enlargement So area of $\Delta\, ORS$ is $3375 \div 18 = 187.5$	M1A1 (3) **[14]**

Notes:

(a) Enlargement for M1
$3\sqrt{2}$ for A1

(b) Answer incorrect, require **CD** for M1

(c) Answer given so require **DB** as shown for B1

(d) Coordinates as shown or written as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 51 \\ 75 \end{pmatrix}$ for each A1

(e) 3375 B1
Divide by theirs for M1